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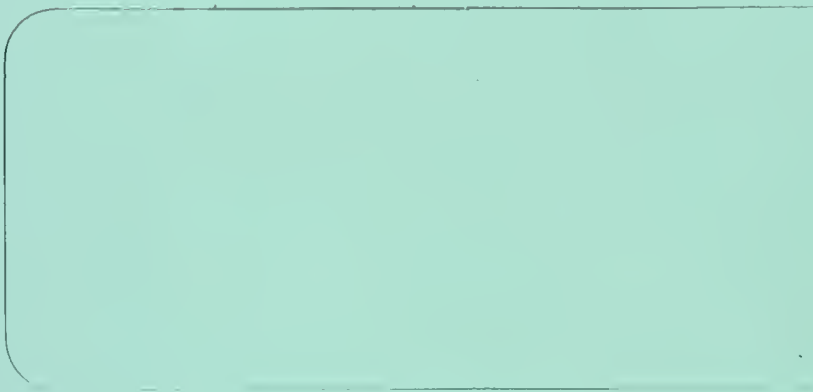
## **Faculty Working Papers**

**EFFECTS OF MEASUREMENT ERRORS ON  
SYSTEMATIC RISK AND PERFORMANCE MEASURE  
OF A PORTFOLIO**

**Cheng F. Lee and Frank C. Jen**

**#369**

**College of Commerce and Business Administration  
University of Illinois at Urbana-Champaign**



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EFFECTS OF MEASUREMENT ERRORS ON SYSTEMATIC RISK  
AND PERFORMANCE MEASURE OF A PORTFOLIO

by

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1. The first part of the paper is devoted to the study of the properties of the function  $f(x)$  defined by the equation

$$f(x) = \int_0^x \frac{1}{1+t^2} dt.$$

2. The second part of the paper is devoted to the study of the properties of the function  $f(x)$  defined by the equation

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3. The third part of the paper is devoted to the study of the properties of the function  $f(x)$  defined by the equation

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# Effects of Measurement Errors on Systematic Risk and Performance Measure of a Portfolio

## I. Introduction

In this paper, we examine the effects of errors in measurement of the two independent variables, return on market ( $R_m$ ) and return on risk-free assets ( $R_f$ ), in the traditional one-factor capital asset pricing model (CAPM). After discussing Sharpe-Lintner's CAPM and both Jensen and Fama's specifications thereof, we review briefly the recent results of Friend and Blume (1970), [FB], Black, Jensen and Scholes (1972) [BJS] and Miller and Scholes (1972) [MS]. In section Two, we first explore possible sources of measurement errors for both  $R_m$  and  $R_f$ , then we specify these errors mathematically and derive analytically their effects on estimates of systematic risk of a security or portfolio,  $\hat{\beta}_j$ , and the Jensen's measure of performance,  $\hat{\alpha}_j$ . In section Three, we derive an analytical expression for the regression coefficient of estimated b's where we estimate the equation  $\hat{\alpha}_j = a + b \hat{\beta}_j$ . The result is then examined to find the conditions under which errors in measurement of  $R_m$  and  $R_f$  can cause b to have a positive or negative value even if the true b is zero. The conditions are then used to examine FB's results and their interpretation. In section IV, an alternative hypothesis testing procedure for the CAPM is examined. We show that the empirical results so derived are also affected by the measurement errors and the sample variation of the systematic risk. The relative advantage between the two different testing hypothesis procedures is then explored. Finally, we comment on the relevance of the result to the popular zero-beta model and indicate areas for further research.

The Sharpe-Lintner form of single-period capital assets pricing model

1. 2. 3.

can be theoretically written as<sup>1)</sup>

$$E(\tilde{R}_j) - R_f = \beta_j [E(\tilde{R}_m) - R_f], \quad (1)$$

where  $\tilde{R}_j$ ,  $R_f$  and  $\tilde{R}_m$  are the rates of return for the  $j$ th asset, the risk-free asset and the market portfolio respectively,  $\beta_j$  is the measure of systematic risk for the  $j$ th asset and  $E$  is the expectation operator.

Empirically, Jensen [1968] specified a time series model as:

$$R_{jt} - R_{Tt} = \alpha_j + \beta_j (R_{mt} - R_{Tt}) + \epsilon_{jt}, \quad (2)$$

where  $\alpha_j$  and  $\beta_j$  are assumed by Jensen to be time invariant,  $R_{Tt}$  is the rate of return for short-term Treasury Bills and  $\epsilon_{jt}$  is assumed to be  $N(0, \sigma_j^2)$  and to be identically and independently distributed over time (i.i.d.). Excess returns for the  $j$ th asset are also assumed to be proportional to excess return for the market portfolio at all times. These assumptions of intertemporal independence and the use of treasury bill rate as a proxy for  $R_f$  enabled Jensen to use time series observations of  $R_{jt}$ ,  $R_{mt}$  and  $R_{Tt}$  to estimate  $\alpha_j$  and  $\beta_j$  for different mutual funds. In addition, Jensen interpreted  $\alpha_j$  as a measure of performance of the  $j$ -th fund. Indeed, Jensen [1968] said, "...if the model is valid, the particular nature of general economic conditions or the particular market condition over the sample or evaluation time period has no effect whatsoever on the measure of performance...".

Jensen's results have been criticized on the ground that assumptions used to derive CAPM are not realistic and that errors of measurement in  $R_m$  and  $R_f$  can affect the results.<sup>2)</sup> Recently, Friend and Blume [1970]

<sup>1)</sup>The form is generally regarded as theoretically superior. See Fama [1968].

<sup>2)</sup>See e.g., Jen [1970], Roll [1969] and Miller and Scholes [1972].





estimated  $\alpha_j$  and  $\beta_j$  for many random portfolios. They then regressed the estimated  $\alpha_j$ 's of different portfolios on the corresponding estimated  $\beta_j$ 's and found the regression coefficient  $b$  to be negative for the entire period 1960-68 and also for the sub-period 1960-1964. They attributed these phenomena to the fact that one of the assumptions used in deriving CAPM, that investors can borrow at the riskless rate for an infinite amount, is violated in practice thus causing the estimated  $\beta_j$ 's to have a negative  $b$ . They further found that for the sub-period 1964-1968,  $b$  is positive. They attributed the latter phenomenon primarily to the unanticipated appreciation in price and not to the violation of the assumption on borrowing. Black, Jensen and Scholes (1972) and Miller and Scholes (1972) regressed average excess return on the estimated  $\beta_j$ 's and concluded the second round regression coefficient is a downward biased estimator of the average excess market rate of return. BJS further attributed these phenomena to the fact that there exists no risk-free rate and advocated the use of zero beta factor.

## II. Effects of Measurement Errors On $\alpha_j$ and $\beta_j$

In this paper we will analyze equation (2), a form of CAPM used empirically by Jensen [1968], Friend and Blume [1970] and Miller and Scholes [1972]. Using hat to denote sample estimate, the estimated regression line can be written as

$$\hat{R}_{jt} - R_{Tt} = \hat{\alpha}_j + \hat{\beta}_j(\hat{R}_{mt} - R_{Tt}), \quad (3)$$

where  $R_{Tt}$  is used as a proxy for  $R_{ft}$ .

We will now derive the properties of  $\hat{\alpha}_j$  and  $\hat{\beta}_j$  when  $R_{mt}$  and  $R_{ft}$  are either with or without errors.





### Measurement Errors on $R_{ft}$

Let us examine first the possible sources of measurement error on  $R_{ft}$ . As has been argued by Roll [1969] and Jen [1970], the treasury bill rate is only a proxy for the risk-free rate. We therefore postulate

$$R_{ft} = R_{Tt} + \tilde{U}_t \quad (4)$$

where  $R_{Tt}$  is treasury bill rates used as a proxy for  $R_{ft}$ ,  $\tilde{U}_t \sim N(0, \sigma_u^2)$  and is i.i.d.

In addition, it is well known that one of the unrealistic assumptions used to derive CAPM is that an investor can borrow freely at the riskless rate. Violation of this assumption has been hypothesized by Friend and Blume to have caused  $\hat{\beta}_j$  to be negatively correlated with  $\hat{\alpha}_j$ .

Allowing for the fact that the borrowing rate is higher than the riskless rate, Brennan [1971] showed that the relationship between return and systematic risk of a capital asset is still linear. He further showed that the only difference between the traditional CAPM and this version is to replace  $R_f$  by  $R_b$ , the latter represents a weighted average of market's lending and borrowing rate. After considering the traditional element of borrow rate, Brennan derived this new form of CAPM:

$$E(R_j) - R_b = \beta_j^* [E(R_m) - R_b]. \quad (5)$$

Following (2) and (5), we can obtain a new regression model as

$$R_{jt} - R_{bt} = \alpha_j^* + \beta_j^* (R_{mt} - R_{bt}) + w_{jt} \quad (6)$$

where  $\alpha_j^*$  and  $\beta_j^*$  are true parameters of the model and  $w_{jt} \sim N(0, \sigma_w^2)$ .

We now postulate that:

$$R_{bt} = R_{Tt} + B + V_{bt}, \quad (7)$$



where  $B$  is positive constant<sup>3)</sup>  $v_{bt}$  is distributed with zero mean and finite variance  $\sigma_b^2$  and is i. i. d.. Substituting (7) and (4) into (6), defining  $e_{bt} = u_{bt} + v_{bt}$ , we have this new theoretical model of CAPM model

$$R_{jt} - (R_{Tt} + B + e_{bt}) = \alpha_j' + \beta_j'[R_{mt} - (R_{Tt} + E + e_{bt})] + w_{jt}, \quad (8)$$

where  $e_{bt} \sim N(0, \sigma_e^2)$ .

It is clear that Jensen's regression equation can be regarded as the correspondent of the form in (8) with error in the measurement of  $R_b$ . Further, the measurement error of  $R_b$  is decomposed into the constant part,  $B$  and the random part  $e_{bt}$ .

Using standard econometric methods we can express  $\hat{\alpha}_j'$  and  $\hat{\beta}_j'$  as estimated parameters in Jensen's equation in (3) when the true model should be that in (8) as

$$\begin{aligned} (a) \quad \hat{\beta}_j' &= \frac{\sum_{t=1}^n \{(r_{jt} - r_{bt} + e_{bt}) (r_{mt} - r_{bt} + e_{bt})\}}{\sum_{t=1}^n (r_{mt} - r_{bt} + e_{bt})^2} \\ (b) \quad \hat{\alpha}_j' &= (\bar{R}_j - \bar{R}_b + B) - \hat{\beta}_j'(\bar{R}_m - \bar{R}_b + B), \end{aligned} \quad (9)$$

where  $r_{jt} = R_{jt} - \bar{R}_j$ ,

$$r_{mt} = R_{mt} - \bar{R}_m,$$

$$r_{bt} = R_{bt} - \bar{R}_b,$$

$\bar{R}_j$ ,  $\bar{R}_m$  and  $\bar{R}_b$  are the arithmetic means of  $R_{jt}$ ,  $R_{mt}$  and  $R_{bt}$  respectively. Following Johnston [1972] and Cramer (1946), the components of the sample estimates  $\hat{\beta}_j'$  and  $\hat{\alpha}_j'$  can be written as<sup>4)</sup>

<sup>3)</sup> Recall  $R_b > R_f$  in the market.

<sup>4)</sup> See the appendix.

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$$\begin{aligned}
 (a) \quad \hat{\beta}_j' &= \beta_j' + \left\{ (1-\beta_j') \frac{\sigma_e^2}{(\sigma_M^2 + \sigma_e^2)} \right\} + S_j \\
 (b) \quad \hat{\alpha}_j' &= \alpha_j' - \left\{ (1-\beta_j') \frac{\sigma_e^2}{(\sigma_M^2 + \sigma_e^2)} + S_j \right\} (\bar{R}_m - \bar{R}_b) \\
 &\quad - \left\{ (1-\beta_j') \frac{\sigma_e^2}{(\sigma_M^2 + \sigma_e^2)} + S_j + (\beta_j' - 1) \right\} B, \quad (10)
 \end{aligned}$$

where  $\beta_j'$  and  $\alpha_j'$  are the population coefficients under the specification of (6);  $\beta_j$  and  $\alpha_j$  are the population coefficients under the specification of (2). Further,  $\beta_j' = \beta_j$  and  $\alpha_j' = \alpha_j + (1 - \beta_j)B$ .  $\sigma_M^2$  is the variance of  $(R_{mt} - R_{bt})$ .  $S_j$  is the sample variation of  $\beta_j$  and is distributed with zero mean and finite variance  $\sigma_s^2$ .

Equation (10a) implies that the sample estimate of the systematic risk can be decomposed into the population value  $\beta_j'$ , sample variation  $S_j$  and the bias caused by the measurement error of  $R_b$ , i.e.  $(1-\beta_j') \frac{\sigma_e^2}{(\sigma_M^2 + \sigma_e^2)}$ . Similarly, as in equation (10b), the sample estimate of the Jensen performance measure  $\alpha_j'$  can be decomposed into three components. The effects of the components other than  $\alpha_j'$  and  $\beta_j'$  are usually ignored. Thus the magnitude of  $\beta_j'$  will affect both the sign and the magnitude of  $E(\hat{\beta}_j')$  and  $E(\hat{\alpha}_j')$ . In general,  $E(\hat{\beta}_j')$  will be larger (or smaller) than  $\beta_j'$  when  $\beta_j'$  is smaller (or larger) than unity.

#### Measurement Errors on $R_{mt}$

We will now investigate the possible sources of measurement error of  $R_{mt}$ . As Roll [1969] has pointed out, "...A New York Stock Exchange (NYSE) average measures  $R_{mt}$  with errors because it only includes a subset of the portfolio...". Indeed, conceptually, we can consider the "market" portfolio to consist of at least three types of assets, i.e. (i) equity asset, (ii) real assets and (iii) debt assets. Equity asset further consists of equities listed on NYSE and American Stock Exchange, and unlisted stocks.



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Hence, using the NYSE stock average only to measure the return of the entire market portfolio ( $R_{mt}$ ) will induce errors in measurement. To take the errors into account, we can postulate the following relationship:

$$R_{mt} = R'_{mt} + \tau + \eta_t \quad (11)$$

where  $R'_{mt}$  is the NYSE average used as a proxy for  $R_{mt}$ , the true rate of return of the market.  $\tau$  and  $\eta_t$  are the constant and random measurement error of  $R_{mt}$  respectively, and  $\eta_t$  is distributed with zero mean and finite variance  $\sigma_\eta^2$ .

Substituting (11) into (8), we have this theoretical form of CAPM:

$$R_{jt} - (R_{Tt} + B + e_{bt}) = \alpha_j'' + \beta_j'' [(R'_{mt} + \tau + \eta_t) - (R_{Tt} + B + e_{bt})] + w_{jt}, \quad (12)$$

where  $(e_{bt} - \eta_t) \sim N(0, \sigma_e^2 + \sigma_\eta^2)$ ,

$$w_{jt} \sim N(0, \sigma_w^2), \quad E(e_{bt}, \eta_t) = 0,$$

$e_t$  and  $\eta_t$  are independent of the true value  $R_{jt}$ ,  $R_{mt}$ , and  $R_{bt}$ ,

$$\alpha_j' = \alpha_j'' \text{ and } \beta_j = \beta_j' = \beta_j''.$$

Equation (12) is the most general form of the one factor CAPM. Since we can generally only observe  $R_{Tt}$  and  $R'_{mt}$  instead of  $R_{bt}$  and  $R_{mt}$ , we can

decompose  $\hat{\beta}_j''$  and  $\hat{\alpha}_j''$  as

$$(a) \quad \hat{\beta}_j'' = \beta_j'' + \psi_j + S_j$$

$$(b) \quad \hat{\alpha}_j'' = \alpha_j'' - (\psi_j + S_j) (\bar{R}_m - \bar{R}_b) - \{\psi_j + S_j + \beta_j\} (\tau - B) + B, \quad (13)$$

$$\text{where } \psi_j = (1 - \beta_j'') (\sigma_e^2 + \sigma_\eta^2) / (\sigma_M^2 + \sigma_e^2 + \sigma_\eta^2). \quad (14)$$

$\hat{\beta}_j''$  and  $\hat{\alpha}_j''$  are the sample estimates of  $\beta_j''$  and  $\alpha_j''$  by employing the observable values of  $R_{jt}$ ,  $R_{Tt}$  and  $R'_{mt}$ .<sup>5)</sup> Note that the sign of  $\psi_j$  depends again on whether  $\beta_j''$  is greater or less than unity.

<sup>5)</sup> When both  $R_b$  and  $R_m$  are measured without error, then equation (13) reduces to a)  $\hat{\beta}_j = \beta_j + S_j$  and  $\hat{\alpha}_j = \alpha_j + S_j(\bar{R}_m - \bar{R}_b)$ . This is the result employed by Friend and Blume (1970) and Black, Jensen and Scholes (1972).

1971-1972  
The following table shows the number of students who have been admitted to the University of Toronto since the year 1971-1972.

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### III. Derivation of Plim $\hat{b}$ Assuming $R_m$ and $R_b$ are Measured With Error

Friend and Blume (1970) have regressed the estimate of Jensen's measures of performance  $\hat{\alpha}_j''$  on the estimate of systematic risk  $\hat{\beta}_j''$  to test whether the one factor CAPM is a useful model. If the measurement errors of  $R_{mt}$  and  $R_{bt}$  do exist as we have postulated, FB's regression can be regarded as having the following form

$$\hat{\alpha}_j'' = a + b\hat{\beta}_j'' + v_j, \quad (15)$$

where  $v_j \sim N(0, \sigma_v^2)$ . After considerable arrangement, and letting the true value of  $b$  equal zero, it can be shown that <sup>6)</sup>

$$\text{plim}_{j \rightarrow \infty} \hat{b} = \left\{ -B + \tau + (\theta k / (1 - \theta)) - \sigma_s^2 (B + k - \tau) / \sigma_{\beta_j}^2 (1 - \theta)^2 \right\} / \left\{ 1 + \sigma_s^2 / \sigma_{\beta_j}^2 (1 - \theta)^2 \right\}, \quad (16)$$

where  $k = \bar{R}_m - \bar{R}_b$ ,  $\sigma_{\beta_j}^2$  = variance of  $\beta_j$ ,  $\sigma_s^2$  = variance of  $S_j$  and

$$\theta = (\sigma_e^2 + \sigma_n^2) / (\sigma_M^2 + \sigma_e^2 + \sigma_n^2).$$

If there exists no measurement errors on both  $R_m$  and  $R_b$ , then (16)

$$\text{reduces to: } \text{plim}_{j \rightarrow \infty} b = -\sigma_s^2 k / (\sigma_{\beta_j}^2 + \sigma_s^2). \quad (17)$$

Equation (17) was actually used by FB to explain the fact that the effect of sampling error on  $\beta_j$  cannot cause  $\hat{b}$  to be negative. However, where only constant measurement errors in both  $R_m$  and  $R_b$  are also considered, (16) reduces to:

$$\text{plim}_{j \rightarrow \infty} \hat{b} = \left\{ -B + \tau - [\sigma_s^2 (B + k - \tau) / \sigma_{\beta_j}^2] \right\} / (1 + \sigma_s^2 / \sigma_{\beta_j}^2). \quad (18)$$

Equation (18) can be used to explain FB's principal finding that  $b < 0$

<sup>6)</sup> See the appendix.

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1. The first step is to identify the problem or question that needs to be answered. This involves understanding the context and the specific requirements of the task.

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since (18) can be re-written as

$$\lim_{j \rightarrow \infty} \hat{b} = \{ -\sigma_s^2 k / (\sigma_{\beta_j}^2 + \sigma_s^2) \} - B + \tau \quad (19)$$

where the first term of equation (19) represents the bias caused by the sample variation of systematic risk in the first round regression,  $-B$  represents the bias caused by the difference between the borrowing and lending rate, and  $\tau$  represents the bias caused by the error in the measurement of  $R_m$ . Hence, if  $-B + \tau$  is significantly negative (positive)  $\hat{b}$  asymptotically will also be negative (positive) when the bias caused by the sampling variation in  $\beta_j$  in the first round regression is very small. Since  $-B = R_f - R_b < 0$ , FB's speculation that the reason that  $\hat{b} < 0$  is due to the divergence of borrowing and lending rate appears as a reasonable proposition.

Finally, when the sampling period is divided into two sub-periods, FB found that  $\hat{b}$  for 1960-64 is negative while that for 1965-68 is positive. FB attributed the latter phenomenon to "unanticipated appreciation" in the market return, though FB did not define the term rigorously. If, following many economists, FB defined unanticipated appreciation as the difference between the realized return to the market and its expected value for the period, the latter cannot cause any bias in  $b$  since the one-factor model takes into account the stochasticity in  $R_m'$ . However, from our equation (19),  $b$  can be positive (negative) if  $-B + \tau$  is positive (negative). Further, if the return to the segment of capital market not represented stocks listed in the New York Stock Exchange has a  $\beta$  greater than unity,  $\tau$  for a sub-period<sup>7)</sup> can be positive (negative) when the realized  $R_m'$  is positive (negative). Hence, even if  $-B < 0$ ,  $b$  for the prosperity period can be positive while that for the recession period will remain negative.

<sup>7)</sup> Strictly speaking, this calls for a multiplicative error-in-variable model (i.e.  $R_m = \theta R_m'$  where  $\theta > 1$ ), an area not yet thoroughly investigated. However, if for a sub-period,  $\tau$  is generally positive (negative)  $E(\tau)$  will then become the "constant" measurement error.  $b$  will then have the same sign as that of  $-B + E(\tau)$  when the effect of random errors is ignored.



1. Introduction

2. Methodology

3. Results and Discussion

4. Conclusion

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#### IV. An Alternative Hypothesis Testing Procedure on $\beta_j$

BJS and MS have also used an alternative hypothesis testing procedure.

After the measurement errors are considered, the null hypothesis used to test the predictive ability of the systematic risk is

$$H_0: c = \tau - B$$

$$d = \bar{R}_m - \bar{R}_b = (\bar{R}'_m - \bar{R}_T) + (\tau - B), \quad (20)$$

where  $c$  and  $d$  are the intercept and the slope of a linear relationship

$$\bar{R}_j = c + d \hat{\beta}_j. \quad (21)$$

The effects of  $\psi_j$  and  $S_j$  on the estimates of both  $c$  and  $d$  can be analyzed using (13a). If we assume that  $\psi_j$  is normally distributed with mean  $(1-\bar{\beta}) (\sigma_e^2 + \sigma_n^2) / (\sigma_M^2 + \sigma_e^2 + \sigma_n^2)$  and finite variance  $\sigma_{\beta_j}^2 [\sigma_e^2 + \sigma_n^2 / (\sigma_M^2 + \sigma_e^2 + \sigma_n^2)]^2$ , then the probability limit of  $\hat{c}$  and  $\hat{d}$  is

$$\begin{aligned} \text{a) } \text{plim}_{j \rightarrow \infty} \hat{d} &= d [(\sigma_M^2 + \sigma_e^2 + \sigma_n^2) / \sigma_M^2] / (1 + \sigma_s^2 / \sigma_{\beta_j}^2) \\ \text{b) } \text{plim}_{j \rightarrow \infty} \hat{c} &= -B + d(1-\bar{\beta}_j) (\sigma_e^2 + \sigma_n^2) / (\sigma_M^2 + \sigma_e^2 + \sigma_n^2) \\ &\quad + \lambda \bar{\beta}_j + \lambda(1-\bar{\beta}_j) (\sigma_e^2 + \sigma_n^2) / (\sigma_M^2 + \sigma_e^2 + \sigma_n^2), \end{aligned} \quad (22)$$

where  $\lambda$  is the asymptotic bias of  $\hat{d}$ , i.e.  $\text{plim } \hat{d} - d$ .

Comparing (16) with (22a), it appears that regressing  $\alpha$  on  $\beta$  will produce a higher bias on estimated parameters than the method described in this section. The reason is that estimate of  $b$  is also affected by constant measurement errors of  $R_b$  and  $R_m$  in addition to the sample variation of  $\beta_j$  and the random measurement error of  $R_b$  and  $R_m$ . However, both  $\bar{R}_m$  and  $\bar{R}_b$  are not observable and  $(\bar{R}'_m - \bar{R}_T)$  is employed to substitute for  $(\bar{R}_m - \bar{R}_b)$  in the null hypothesis. Therefore, the hypothesis to be tested is not precisely stated at all. Hence, even if the bias in the estimated



The following is a list of the names of the persons who have been appointed to the various positions in the Department of the Interior, under the act of March 3, 1879, entitled "An Act to provide for the better management of the public lands, and for other purposes."

parameters are smaller, it does not mean that the test is statistically more powerful.

The relationship between  $b$  and  $d$ , and the relative magnitude between the coefficient of determination for (15) and (21) will be investigated as follows:

(i) From the definition of the regression slope, we have

$$\begin{aligned}
 \hat{b} &= \frac{\text{Cov}(\hat{\alpha}_j, \hat{\beta}_j)}{\text{Var } \hat{\beta}_j} \\
 &= \frac{\text{Cov}\{[(\bar{R}_j - \bar{R}_T) - \hat{\beta}_j(\bar{R}_m - \bar{R}_T)], \hat{\beta}_j\}}{\text{Var}(\hat{\beta}_j)} \\
 &= \frac{\text{Cov}[(\bar{R}_j - \bar{R}_T), \hat{\beta}_j]}{\text{Var}(\hat{\beta}_j)} - \frac{\text{Var}(\hat{\beta}_j)(\bar{R}_m - \bar{R}_T)}{\text{Var}(\hat{\beta}_j)} \\
 &= \hat{d} - (\bar{R}_m - \bar{R}_T)
 \end{aligned} \tag{23}$$

(ii) The coefficient of determination for (15) can be derived as:<sup>8)</sup>

$$\begin{aligned}
 R_1 &= \frac{\text{Cov}(x, y) - k \text{Var}(y)}{\sqrt{(\text{Var } y)(\text{Var } x + k^2 \text{Var}(y) - 2k \text{Cov}(x, y))}} \\
 &= \frac{\text{Cov}(x, y)}{\sqrt{(\text{Var } y)(\text{Var } x)}} - k \sqrt{\frac{\text{Var } y}{\text{Var } x}} \left[ \frac{\text{Var } x}{\sqrt{\text{Var } x + k^2 \text{Var } y - 2k \text{Cov}(x, y)}} \right]
 \end{aligned} \tag{24}$$

where  $x = \bar{R}_j - \bar{R}_T$ ,

$k = \bar{R}_m - \bar{R}_T$ ,  $y = \hat{\beta}_j$ .

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<sup>8)</sup> In a simple regression, Theil [12] has shown that the coefficient of determination is identical to the correlation coefficient between the explained and the explanatory variables.

The following is a list of the names of the persons who have been  
 elected to the office of the President of the United States since  
 the year 1789. The names are given in the order in which they  
 were elected, and the year of their election is given in parentheses.  
 The names are given in the order in which they were elected, and  
 the year of their election is given in parentheses.

George Washington (1789)  
 John Adams (1797)  
 Thomas Jefferson (1801)

James Madison (1809)  
 James Monroe (1817)

John Quincy Adams (1825)  
 Andrew Jackson (1829)

Martin Van Buren (1837)  
 William Henry Harrison (1840)

Franklin Pierce (1853)  
 James Buchanan (1857)

Abraham Lincoln (1861)  
 Andrew Johnson (1865)

Ulysses S. Grant (1869)  
 Rutherford B. Hayes (1877)  
 James A. Garfield (1881)  
 Chester A. Arthur (1881)  
 Grover Cleveland (1885)  
 Benjamin Harrison (1889)  
 William McKinley (1897)  
 Theodore Roosevelt (1901)  
 William Howard Taft (1909)  
 Woodrow Wilson (1913)  
 Warren G. Harding (1921)  
 Calvin Coolidge (1923)  
 Herbert Hoover (1929)  
 Franklin D. Roosevelt (1933)  
 Harry S. Truman (1945)  
 Dwight D. Eisenhower (1953)  
 John F. Kennedy (1961)  
 Lyndon B. Johnson (1963)  
 Richard M. Nixon (1969)  
 Gerald R. Ford (1974)  
 Jimmy Carter (1977)  
 Ronald Reagan (1981)  
 George H. W. Bush (1989)  
 Bill Clinton (1993)  
 George W. Bush (2001)  
 Barack Obama (2009)  
 Donald Trump (2017)

Similarly, the coefficient of determination for (21) can be defined as

$$R_2 = \frac{\text{Cov}(x, y)}{\sqrt{(\text{Var } y)(\text{Var } x)}} \quad (25)$$

By comparing  $R_1$  with  $R_2$ , it is found that the coefficient of determination of (15) is determined by three different components, i.e., (i) the coefficient of determination of (21), (ii)  $k\sqrt{\text{Var } y / \text{Var } x}$  and (iii)  $[\text{Var } x / \text{Var } x + k^2 \text{Var } y - 2k \text{Cov}(x, y)]^{1/2}$ . If  $\text{Cov}(x, y)$  approaches zero, then  $R_2^2$  will approach  $k^2 \text{Var } y / \text{Var } x + k^2 \text{Var } y$ .<sup>9)</sup> Incidentally, (22a) is the general case of the result derived by Black, Jensen and Scholes [1] who considered only the effect of sampling variation.

Jensen (1968) has employed the sample distribution of  $\hat{\alpha}_j''$  ( $j=1\dots m$ ) to show the usefulness of Jensen performance measures. Black, Jensen and Scholes employed the similar concept to do the time series test of CAPM. In addition, they claimed that the time series test is free from the measurement error of the systematic risk. From (13b), however, we have demonstrated that the sample distribution of  $\hat{\alpha}_j''$  is affected by: (1) the sample variation of  $\beta_j$ , and (2) the measurement error of both  $R_b$  and  $R_m$ .<sup>10)</sup>

## V. Conclusion

To sum up, our analysis indicated that both the cross sectional tests and the time series test of the CAPM are affected by the measurement error of  $R_b$  and  $R_m$  and the sample variation of the systematic risk. Hence, interpretation of the empirical results of CAPM should be done with extreme care. Further, the use of the ordinary least squares method should also

<sup>9)</sup> Using random portfolio data, Friend and Blume (1970) have calculated  $R_1^2$  and Blume and Friend (1973) have calculated  $R_2^2$  for similar time periods.

<sup>10)</sup> Since  $\hat{\alpha}_j''$  includes the components  $\psi_j$ ,  $S_j$ ,  $B$  and  $\tau$  in addition to the true value of Jensen measure  $\alpha_j$ .



be reconsidered and the error-in-variable approach used wherever possible.

As to areas for further research, it is clear that measurement errors will also affect estimates derived from the two-factor models since the one-factor errors-in-variable model presented above is conceptually quite similar to the two factor model.<sup>11)</sup> Finally, there are problems associated with nonstationarity of  $\beta$ 's which may mean that the current CAPM is not correctly specified in the first place. [see Galai and Masulis (7)]

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<sup>11)</sup>From Black, Jensen and Scholes (1972) and the results of this paper, one can show that the estimate of so-called zero beta factor is not free from the measurement error also even if BJS's approach may have reduced sample variations. New procedures will however have to be derived to eliminate these errors in practice.

1. The first part of the document discusses the importance of maintaining accurate records of all transactions. It emphasizes that proper record-keeping is essential for the integrity of the financial system and for the ability to detect and prevent fraud. The text also mentions the need for regular audits and the role of independent auditors in ensuring the reliability of the data.

2. The second part of the document focuses on the challenges faced by organizations in implementing effective internal controls. It highlights the complexity of modern business environments and the need for a robust framework of controls to manage risks. The text suggests that organizations should adopt a risk-based approach to internal control design and implementation, focusing on the most significant risks to the organization's objectives.

3. The third part of the document discusses the importance of transparency and accountability in financial reporting. It notes that stakeholders, including investors, creditors, and the public, rely on the information provided in financial statements to make informed decisions. The text stresses the need for organizations to provide clear, concise, and reliable information, and to be held accountable for the accuracy of their reports.

## CONCLUSION

In conclusion, the document underscores the critical role of internal controls and financial reporting in ensuring the integrity and reliability of an organization's financial information. It calls for a commitment to high standards of practice, regular monitoring, and continuous improvement of internal control systems. The text also emphasizes the importance of a strong corporate culture that values transparency and accountability, and the role of leadership in setting the tone at the top.



## Appendix

### (A) The decomposition of the estimated regression coefficients

The sample estimation of either slope or intercept can be decomposed into true population part and sample variation part if data are observed without error. However, if the data are observed with error, some additional components should be included in the estimated regression coefficients as indicated in this paper.

### (B) The derivation of equation (10)

Taking the probability limit of equation (9a), we have

$$\begin{aligned} \text{plim}_{t \rightarrow \infty} \hat{\beta}_j' &= (\beta_j' + \sigma_e^2) / (\sigma_M^2 + \sigma_e^2) \\ &= \beta_j' + (1 - \beta_j') \sigma_e^2 / (\sigma_M^2 + \sigma_e^2) \end{aligned} \quad (a)$$

and since

$$\hat{\beta}_j' = \text{plim}_{t \rightarrow \infty} \hat{\beta}_j' + S_j \quad (b)$$

therefore, after substituting (a) into (b) we have (10a). Similarly, we know that

$$\begin{aligned} \hat{\alpha}_j' &= \text{plim}_{t \rightarrow \infty} \hat{\alpha}_j' - S_j (\bar{R}_m - \bar{R}_b + B) \\ &= (\bar{R}_j - \bar{R}_b + B) - \text{plim}_{t \rightarrow \infty} \hat{\beta}_j' (\bar{R}_m - \bar{R}_b + B) - S_j (\bar{R}_m - \bar{R}_b + B) \end{aligned} \quad (c)$$

substituting (a) into (c), we have (10b).

### (C) The derivation of equation (16)

From equation (13), we can obtain

$$\begin{aligned} \hat{\alpha}_j'' &= (y + B) - (k + B - \tau) (\beta_j + \psi_j + S_j) \\ \hat{\beta}_j'' &= \beta_j + \psi_j + S_j, \end{aligned} \quad (a)$$

where  $y = \bar{R}_j - \bar{R}_b$ ,  $k = \bar{R}_m - \bar{R}_b$ .

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It is well-known that the sample estimation of  $b$  in (15) can be written as

$$\hat{b} = \frac{\sum_{j=1}^h (\hat{\alpha}_j'' - \bar{\alpha}_j'') (\hat{\beta}_j'' - \bar{\beta}_j'')}{\sum_{j=1}^h (\hat{\beta}_j'' - \bar{\beta}_j'')^2} \quad (b)$$

where  $h$  is the number of the portfolio. Taking the probability limit of  $\hat{b}$ , we have

$$\text{plim}_{j \rightarrow \infty} \hat{b} = \frac{\text{Cov}(\hat{\alpha}_j'', \hat{\beta}_j'')}{\text{Var}(\hat{\beta}_j'')} \quad (c)$$

Based upon (a) and the definition of variance and covariance, we have

$$\text{Var}(\hat{\beta}_j'') = E(\beta_j'' + \psi_j + S_j)^2 - [E(\beta_j'' + \psi_j + S_j)]^2 \quad (d)$$

$$\begin{aligned} \text{Cov}(\hat{\alpha}_j'', \hat{\beta}_j'') &= E[(y+B)(\beta_j'' + \psi_j + S_j) - (k+B-\tau)(\beta_j'' + \psi_j + S_j)^2] \\ &\quad - E[(y+B) - (k+B-\tau)(\beta_j'' + \psi_j + S_j)] E[\beta_j'' + \psi_j + S_j] \end{aligned} \quad (e)$$

After combining (c), (d) and (e) and letting the true value of  $b$  equal zero, we will obtain (16).

11. Let  $f(x) = x^2 + 1$  and  $g(x) = x^2 - 1$ . Find  $(f \circ g)(x)$  and  $(g \circ f)(x)$ .

$$(f \circ g)(x) = f(g(x)) = f(x^2 - 1) = (x^2 - 1)^2 + 1 = x^4 - 2x^2 + 1 + 1 = x^4 - 2x^2 + 2$$

$$(g \circ f)(x) = g(f(x)) = g(x^2 + 1) = (x^2 + 1)^2 - 1 = x^4 + 2x^2 + 1 - 1 = x^4 + 2x^2$$

12. Let  $f(x) = x^2 + 1$  and  $g(x) = x^2 - 1$ . Find  $(f \circ g)(x)$  and  $(g \circ f)(x)$ .

$$(f \circ g)(x) = f(g(x)) = f(x^2 - 1) = (x^2 - 1)^2 + 1 = x^4 - 2x^2 + 1 + 1 = x^4 - 2x^2 + 2$$

$$(g \circ f)(x) = g(f(x)) = g(x^2 + 1) = (x^2 + 1)^2 - 1 = x^4 + 2x^2 + 1 - 1 = x^4 + 2x^2$$

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$$(f \circ g)(x) = f(g(x)) = f(x^2 - 1) = (x^2 - 1)^2 + 1 = x^4 - 2x^2 + 1 + 1 = x^4 - 2x^2 + 2$$

$$(g \circ f)(x) = g(f(x)) = g(x^2 + 1) = (x^2 + 1)^2 - 1 = x^4 + 2x^2 + 1 - 1 = x^4 + 2x^2$$

14. Let  $f(x) = x^2 + 1$  and  $g(x) = x^2 - 1$ . Find  $(f \circ g)(x)$  and  $(g \circ f)(x)$ .

15. Let  $f(x) = x^2 + 1$  and  $g(x) = x^2 - 1$ . Find  $(f \circ g)(x)$  and  $(g \circ f)(x)$ .

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